ADMITTANCE OF INFINITE AND FINITE CYLINDRICAL METALLIC ANTENNAS.

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A travelling wave theory of cylindrical antennas of finite thickness is considered. Special emphasis is given to the so-called gap problem and the end problem. The specific antenna studied is a monopole over a groundplane excited by a coaxial line. The geometry is shown in Fig. 1 together with the mathematical model of the excitation, a magnetic ring current at $r = r_s$ where r_s is allowed to be greater than a, the radius of the antenna. r_s is approximately equal to $\frac{1}{2}(a+b)$.

The conductance, G_∞, of the infinite antenna is found by integration of the far field. The numerical results indicate that the conductance decreases when b/a increases, the effect is of importance when k_ob is greater than 0.5. The susceptance corresponds to the capacitance between the cylindrical wire and the groundplane and is not determined theoretically; an approximate value for narrow gap ($b-a \ll a$) is

$$B_\infty = -\frac{4k_oa}{\zeta_o}\ \ln\frac{\lambda e}{4\Gamma(b-a)}\ , \qquad (1)$$

($\zeta_o = 120\pi$ ohms, $\Gamma = 1.781 \ldots$, $e = 2.718 \ldots$)
which illustrates the proper limiting behaviour, B_∞ tends to zero when $k_oa \to 0$ for fixed gap and tends to infinity for $b \to a$ for fixed k_oa. Accurate values derived from experiment are shown in Fig. 2 and will be commented on later.

The amplitude of the travelling wave current vs. distance is given in terms of a wave admittance, $Y_w(z/\ell)$, which depends only on the thickness parameter

$$\ell = \frac{k_oa^2\Gamma}{2}\ . \qquad (2)$$

The wave is a quasi-cylindrical wave slightly damped due to radiation losses. A universal curve $Y_w(x)$ has been found numerically for $x > 1$.

The infinite antenna is made finite by absorption of the outgoing wave at the end, and the radiation pattern is found approximately. The admittance of this antenna is of course equal to $Y_\infty = G_\infty + iB_\infty$. This antenna (Altshuler (1961)) is of practical interest but here it also serves as an intermediate step towards the third type, the antenna with reflections from the end. The ends are treated as effective gaps with an end admittance, Y_e, which depends on k_oa and on the geometry of the end. The end admittance Y_e for a thin-walled tube has been found by Hallèn (1956). The end gives rise to reflected waves which are partly transmitted into the feeding coaxial line and partly reflected back towards the end. By adding upon the effect of the multiply scattered waves the steady state solution for the admittance is found to be

$$Y = Y_\infty - \frac{2Y_w^{\,2}(L/\ell)\, e^{i2k_oL}}{Y_e + Y_w(2L/\ell)\, e^{i2k_oL}} \tag{3}$$

where L is the length of the monopole.

For an antenna with unknown Y_∞ and Y_e but Y known from measurements for two values of L eq. 3 can be considered as an equation from which Y_∞ and Y_e can be determined. The experimental results by Hartig (King (1956)) for $kL = \frac{1}{2}\pi$ and π have been used in this manner and the results are shown in Figs. 2 and 3.

Almost all existing theories use a deltafunction generator with a corresponding singularity in susceptance which is circumvented in a more or less arbitrary manner. Duncan (1962) chooses to extrapolate the slowly varying part of the current right into the gap and Hallèn (1953) uses the current at a distance from the gap approximately equal to the radius. All these theories imply a gap of a certain (but previously unknown) size and for gaps of a different size the theories will disagree with experiment. King (1955) has made a correction for large gaps but not for small gaps. King-Middletons second-order theory implies a ratio of b/a about 2.

From Fig. 3 it is noted that the end conductance is larger for an end with a hemispherical cap than for a tube ending and the difference is increasing with k_oa. The susceptance of the rounded end is smaller than the susceptance of the sharp-edged end.

References:

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Hallèn, E. (1953), Elektricitetslära, Stockholm.

Hallèn, E. (1956), Exact treatment of antenna current wave reflection at the end of a tube-shaped cylindrical antenna, IRE Trans.Ant. Prop., AP-4, No. 3, 479-491.

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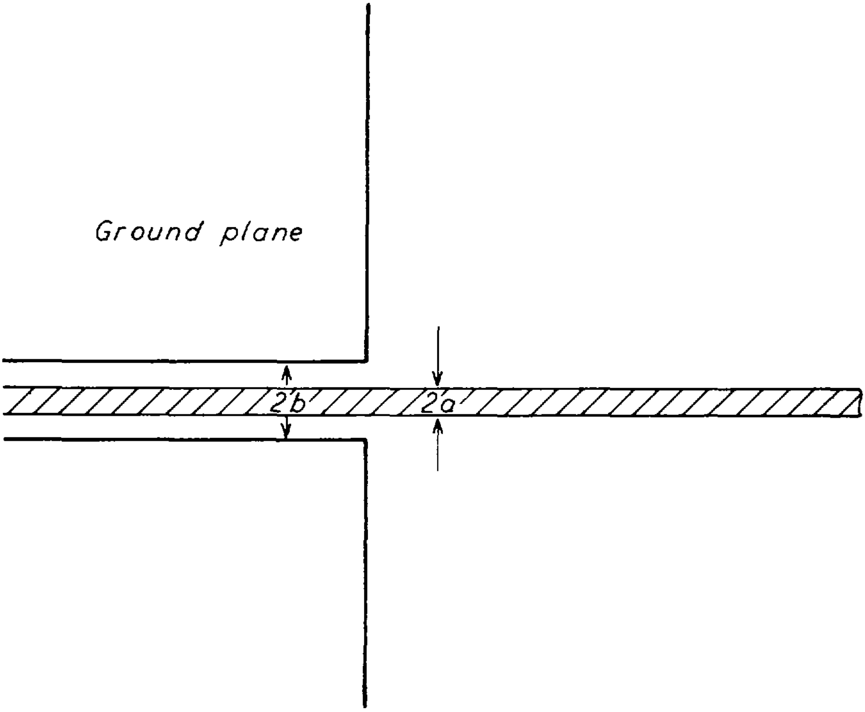
Fig. 1. Models of infinite antenna. a)Physical model is a semi-infinite monopole excited by a coaxial line. b) Mathematical model is an infinite cylinder excited by a magnetic ring current at radius r_s.

Fig. 2. Admittance of infinite antenna. The points are experimentally derived values. For the susceptance the theoretical curves are those of Hallén (1953) and Duncan (1962).

Fig. 3. End admittance. The experimentally derived values for an end with hemispherical cap are compared with theoretical values of Hallén (1956) for a thin-walled tube.



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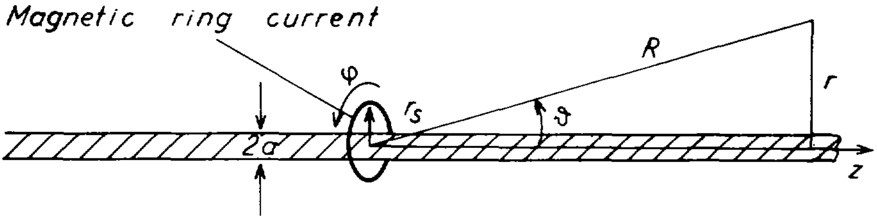
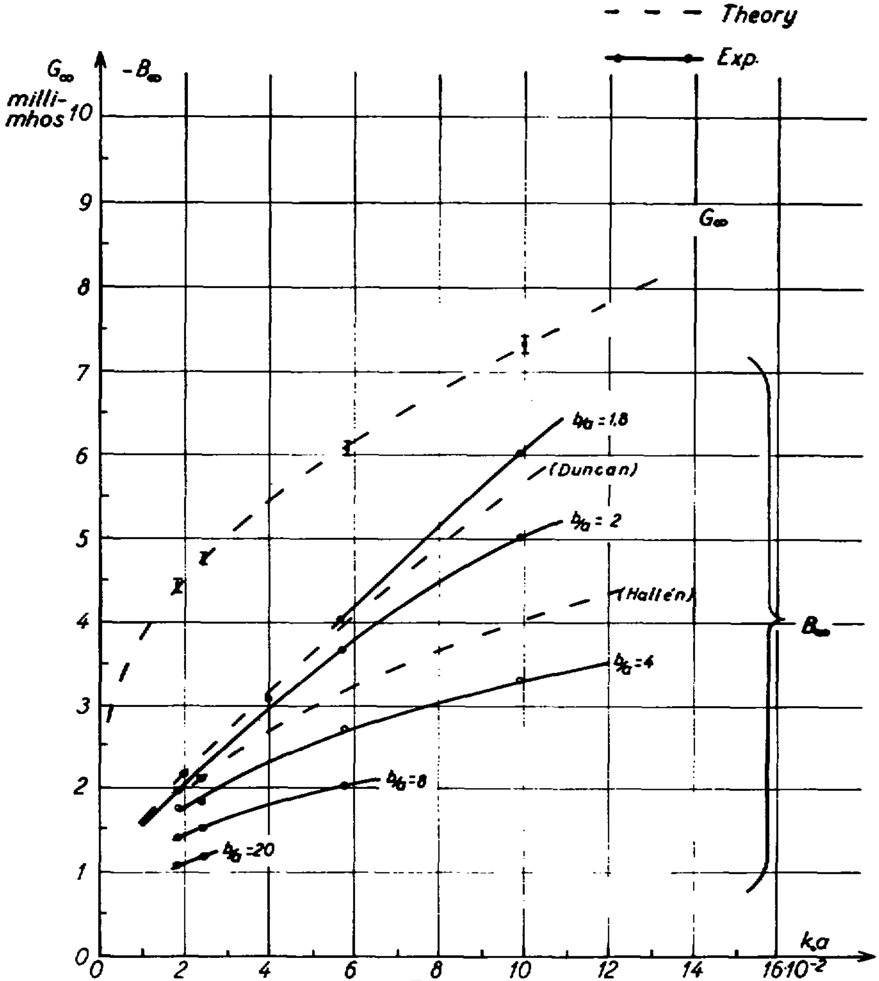
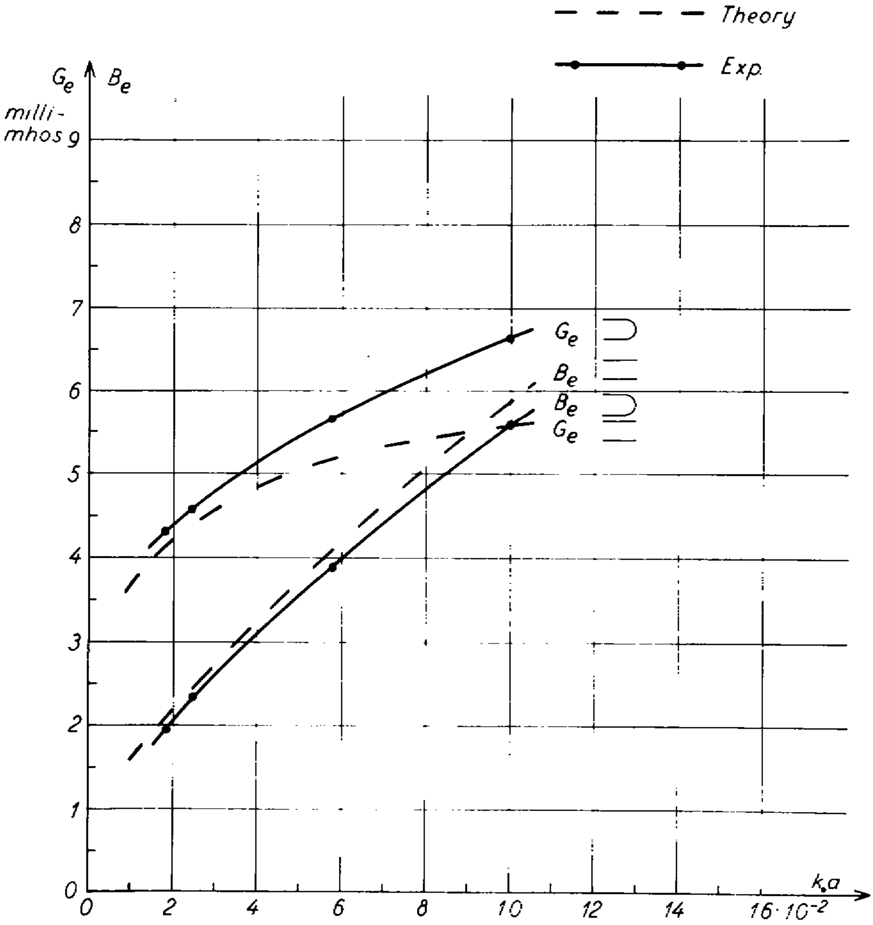




Theory
Exp.
G_∞
$-B_\infty$
milli-mhos
10
9
8
7
6
5
4
3
2
1
0
G_∞
$b/a = 1.8$
(Duncan)
$b/a = 2$
(Hallén)
$b/a = 4$
$b/a = 8$
$b/a = 20$
B_∞
k_0a
0
2
4
6
8
10
12
14
$16 \cdot 10^{-2}$

Fig 2


Theory
Exp.
G_e B_e
milli-mhos
9
8
7
6
5
4
3
2
1
0
0
2
4
6
8
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12
14
$16 \cdot 10^{-2}$
$k_0 a$
G_e
B_e
B_e
G_e